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Idempotents in Group Algebras and Exceptional Characters

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Exceptional characters are irreducible characters of a finite group associated with characters of a subgroup in a certain way. The theory of exceptional characters gives a formula for the differences $\chi_i - \chi_j$ of exceptional characters, rather than a formula for the characters themselves. Often information about the other characters of the group is used to pin down the exceptional characters. In this paper it is shown, by evaluating characters on certain group algebra idempotents, that the exceptional character χ_i is determined by structure constants associated with those conjugacy classes of the group contained in the support of $\chi_i - \chi_j$. For the most part, the results of this paper have proofs employing little more than the orthogonality relations for group characters.

1. PRELIMINARIES

Throughout this paper, G will be a finite group whose identity is denoted by 1, having conjugacy classes $K_0 = \{1\}, K_1, \ldots, K_s$, with fixed class representatives $g_i \in K_i$. The sum in the complex group algebra $\mathfrak{C}G$ of the elements in K_i is denoted by C_i. These class sums form a basis for the center of $\mathfrak{C}G$. The structure constants c_{ijk} defined by $C_iC_j = \sum_k c_{ijk}C_k$ are integers satisfying

$$c_{ijk} = |\{(x, y) \mid x \in K_i, y \in K_j, xy = g_k\}| \tag{1.1}$$

which is independent of the choice of the class representative g_k. Let $\chi_0 = 1_G, \chi_1, \ldots, \chi_s$ be the irreducible characters of G. The degree $\chi_i(1)$ of χ_i will be denoted by z_i. There is a well-known formula [4, (2.15)] giving c_{ijk} in terms of the irreducible characters and the orders of the conjugacy classes. Conversely, Burnside [1, Chapter XV] showed how the irreducible

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characters might be obtained from a knowledge of all the structure constants. We will show in this paper how exceptional characters can be obtained from a knowledge of a limited number of structure constants.

We proceed now to give some formulas needed in the later sections. References include [3, Chapter V], [4, Chapter I], and a paper of Janusz [6].

The central idempotent in $\mathfrak{C}G$ associated with an irreducible character χ is denoted by $e(\chi)$. This idempotent is given by

$$e(\chi) = \chi(1) \mid G \mid^{-1} \sum_{g \in G} \chi(g) g^{-1}. \tag{1.2}$$

If χ_i and χ_j are distinct irreducible characters of G then $e(\chi_i)$ and $e(\chi_j)$ are orthogonal idempotents. This fact is equivalent to the following generalized orthogonality relation:

$$\mid G \mid^{-1} \sum_{g \in G} \chi_i(g) \chi_j(g^{-1}h) = \delta_{ij}\chi_i(h)/z_i \tag{1.3}$$

where δ_{ij} is the Kronecker delta and h is a fixed but arbitrary element of G. If $h = 1$ in (1.3) we get the standard orthogonality relation

$$(\chi_i , \chi_j) = \mid G \mid^{-1} \sum \chi_i(g) \chi_j(g^{-1}) = \delta_{ij} . \tag{1.4}$$

In general, for class functions σ, we write

$$\| \sigma \|^2 = (\sigma, \sigma) = \mid G \mid^{-1} \sum \sigma(g) \overline{\sigma(g)}.$$

Formulas (1.2) and (1.4) imply

$$\chi_j(e(\chi_i)) = z_i\delta_{ij} . \tag{1.5}$$

We say that a primitive idempotent e in $\mathfrak{C}G$ affords the irreducible character χ, if the module $\mathfrak{C}Ge$ affords a representation having character χ. The central idempotent $e(\chi)$ is the sum of $\chi(1)$ mutually orthogonal primitive idempotents each affording χ. If f is an arbitrary idempotent in $\mathfrak{C}G$ such that $\mathfrak{C}Gf$ affords the character θ, then it can be shown that

$$m = \chi(f) = (\chi, \theta) = \chi(e(\chi)f) \tag{1.6}$$

is the number of times the irreducible character χ appears in the character θ. In this case the character $m\chi$ is afforded by $e(\chi)f$.

For additional properties of characters used below, especially those of induced characters and exceptional characters, the reader may consult [3], [4], or [5].

The following lemma, a consequence of the generalized orthogonality relation (1.3), relates characters to structure constants and will be used several times. A generalized character is an ordinary character or a difference of ordinary characters, hence a linear combination of irreducible characters with rational integral coefficients.

LEMMA 1. *Let* $\theta = \sum_{r=0}^{s} m_r\chi_r$ *be a generalized character of* G. *Then*

$$\sum_{g\in G} \theta(g)\,\theta(g^{-1}g_k) = |G| \sum_{r=0}^{s} z_r^{-1} m_r^2 \chi_r(g_k) = \sum_{i,j=0}^{s} \theta(g_i)\,\theta(g_j)\,c_{ijk}\,.$$

Proof. From the definition of θ and (1.3) we have

$$\begin{aligned}\sum_{g\in G} \theta(g)\,\theta(g^{-1}g_k) &= \sum_{r,t=0}^{s} m_r m_t \sum_{g\in G} \chi_r(g)\,\chi_t(g^{-1}g_k) \\ &= \sum_{r,t=0}^{s} m_r m_t\, |G|\, \delta_{rt}\chi_r(g_k)/z_r\end{aligned}$$

and the first equality holds. We also have

$$\sum_{g\in G} \theta(g)\,\theta(g^{-1}g_k) = \sum_{i=0}^{s} \theta(g_i) \sum_{g\in K_i} \theta(g^{-1}g_k).$$

Now as g ranges over K_i, the element $g^{-1}g_k$ will be some $y \in K_j$ if and only if $gy = g_k$ and, by (1.1), this will happen exactly c_{ijk} times. Thus our formula may be written as

$$\sum_{i=0}^{s} \theta(g_i) \sum_{j=0}^{s} \theta(g_j) c_{ijk}$$

and the second equality holds.

2. EXCEPTIONAL CHARACTERS AND CENTRAL IDEMPOTENTS

THEOREM 1. *Suppose* χ_1 *and* χ_2 *are distinct irreducible characters of* G *having the same degree* z. *Let the element* f *in the group algebra be defined by*

$$f = |G|^{-1} \sum (\chi_1 - \chi_2)(g)\, g^{-1}$$

and let σ *be the function on* G *satisfying*

$$f^2 = |G|^{-1} \sum \sigma(g)\, g^{-1}.$$

Then f is in the center of the group algebra, σ is a class function, and the following equalities hold:

(i) $e(\chi_1) - e(\chi_2) = zf$

(ii) $e(\chi_1) = \frac{1}{2}(zf + z^2f^2)$

(iii) $z = \sqrt{2}/\|\sigma\|$.

Proof. The element f is central since its coefficients are constant on conjugacy classes. Hence f^2 is also central, so that σ is a class function. If we use (1.2) to express $e(\chi_1) - e(\chi_2)$ and factor out the common degree z we obtain (i). Now since zf is the difference of orthogonal idempotents, $(zf)^2$ is the sum of those idempotents and from this (ii) follows. To get (iii) we again use $e(\chi_1) + e(\chi_2) = z^2f^2$ and the definition of σ to obtain

$$z\,|G|^{-1} \sum (\chi_1 + \chi_2)(g)\, g^{-1} = z^2\,|G|^{-1} \sum \sigma(g)\, g^{-1}.$$

Hence as functions on G, $\chi_1 + \chi_2 = z\sigma$ and we have

$$2 = \|\chi_1 + \chi_2\|^2 = \|z\sigma\|^2 = z^2\|\sigma\|^2.$$

Solving for z completes the proof.

We note that if $\chi_1 - \chi_2 = (\psi_1 - \psi_2)^G$ for irreducible characters ψ_i of a subgroup H of G then the element f in Theorem 1 can be given by

$$f = (\psi(1)[G:H])^{-1} \sum_{k=1}^{t} x_k^{-1}(e(\psi_1) - e(\psi_2))x_k$$

where $x_1, \ldots, x_t$ is a full set of coset representatives of H in G. Moreover if $\psi_1 - \psi_2$ has support in a T.I. set in G (as is the case when χ_1, χ_2 are exceptional characters), then the summation is over elements of the group algebra with pairwise disjoint support. To find the common degree z and the values $\chi_1(g)$ via Theorem 1, one needs to know the function σ. This depends on being able to square the element f in the center of the group algebra, which in turn requires knowledge of structure constants. Explicit formulas giving z and χ_1 in terms of $\chi_1 - \chi_2$ and certain structure constants are given in Theorem 3.

Our next result, a consequence of Lemma 1, gives a new formula relating character degrees with structure constants.

THEOREM 2. *Let $\theta = \sum_{r=0}^{s} m_r\chi_r$ be a generalized character of G. Then*

$$\sum_{r=0}^{s} \frac{m_r^{\,4}}{z_r^{\,2}} = |G|^{-3} \sum_{k=0}^{s} |K_k| \left| \sum_{i,j=0}^{s} \theta(g_i)\,\theta(g_j)c_{ijk} \right|^2.$$

Proof. By Lemma 1, the right side of this equation is

$$|G|^{-3}\sum_{k=0}^{s}|K_k|\,|G|^2\left|\sum_{r=0}^{s} z_r^{-1}m_r^{\,2}\chi_r(g_k)\right|^2$$

$$=|G|^{-1}\sum_{k=0}^{s}|K_k|\sum_{r,t=0}^{s} z_r^{-1}z_t^{-1}m_r^{\,2}m_t^{\,2}\chi_r(g_k)\,\overline{\chi_t(g_k)}$$

$$=|G|^{-1}\sum_{r,t=0}^{s} z_r^{-1}z_t^{-1}m_r^{\,2}m_t^{\,2}\,|G|(\chi_r,\chi_t).$$

Now since $(\chi_r,\chi_t)=\delta_{rt}$, the proof is complete.

We now give the main theorem of this section.

THEOREM 3. *Let χ_1 and χ_2 be as in Theorem 1. Let $K_1,\ldots,K_w$ be the conjugacy classes in the support of $\chi_1-\chi_2$. Then*

$$z=\left[2|G|^3\left(\sum_{k=0}^{s}|K_k|\left|\sum_{i,j=1}^{w}(\chi_1-\chi_2)(g_i)(\chi_1-\chi_2)(g_j)c_{ijk}\right|^2\right)^{-1}\right]^{1/2} \tag{2.1}$$

and

$$\chi_1(g_k)=\tfrac{1}{2}\Big((\chi_1-\chi_2)(g_k)+z|G|^{-1}\sum_{i,j=1}^{w}(\chi_1-\chi_2)(g_i)(\chi_1-\chi_2)(g_j)c_{ijk}\Big). \tag{2.2}$$

Proof. Let $\theta=\chi_1-\chi_2$ and solve for z in Theorem 2. This yields (2.1). This formula also follows from Theorem 1 (iii) by an application of Lemma 1. Formula (2.2) may be interpreted as a restatement of Theorem 1 (iii) and can be proved directly using Lemma 1 and the fact that

$$\chi_1(g_k)=\tfrac{1}{2}((\chi_1-\chi_2)(g_k)+(\chi_1+\chi_2)(g_k)).$$

In both (2.1) and (2.2) the indices i and j need run only from 1 to w since for other classes the summands would be zero.

Computation of (2.1) might be eased somewhat by nothing that for $k=0$, the summand is just $(|G|\,\|\chi_1-\chi_2\|^2)^2=4|G|^2$. We see that the structure constants appearing in (2.1) and (2.2) are just those associated with products of class sums for classes in the support of $\chi_1-\chi_2$. Suppose $\chi_1-\chi_2=(\psi_1-\psi_2)^G$ where χ_1 and χ_2 are exceptional characters associated with the irreducible characters ψ_1 and ψ_2 on a subgroup H, the normalizer of a T.I. set (c.f. [4, Section 23] or [5, Section 4.4]). Then the classes in the support of $\chi_1-\chi_2$, called *special classes*, all meet the support of $\psi_1-\psi_2$ in H and we may assume $g_i\in[K_i\cap\text{support}(\psi_1-\psi_2)]$, $i=1,\ldots,w$. We have by the theory of exceptional characters, $(\chi_1-\chi_2)(g_i)=(\psi_1-\psi_2)(g_i)$. The appropriate substitutions in formulas (2.1) and (2.2) yield the following

COROLLARY. *The values of exceptional characters may be determined from a knowledge of* $|G|, |K_1|, \ldots, |K_s|$, *the appropriate characters on the normalizer of the* T.I. *set, and the structure constants arising from products of special class sums.*

Formulas analogous to (2.1) and (2.2) exist for characters obtained from coherent sets of characters by lifting operators other than induction. Specifically for π induction as developed by Feit and Thompson, Dade, Leonard and McKelvey, and Reynolds (see [7] for a survey and a list of references on exceptional characters and generalizations), our formulas hold if we replace the structure constants by certain sums of structure constants associated with products of π-section sums in the center of the group algebra.

3. Exceptional Characters and Primitive Idempotents

THEOREM 4. *Suppose* χ_1 *and* χ_2 *are irreducible characters of* G *of the same degree* z *such that* $\chi_1 - \chi_2$ *has support on the conjugacy classes* $K_1, \ldots, K_w$. *Let* ψ *be an irreducible character on a subgroup* H *of* G *such that* $(\chi_1 + \chi_2, \psi^G)$ *is nonzero and let* $f = \sum_{h\in H} \tau(h)_h$ *be a primitive idempotent in* $\mathfrak{C}H$ *affording* ψ. *For each conjugacy class of* G *let* $S_k = K_k \cap \text{support}(\tau)$,

Let $S = \{k \mid S_k \neq \phi\}$. *Then*

$$z = (\chi_1 + \chi_2, \psi^G) \mid G \mid \Big/ \left[\sum_{i,j=1}^{w} (\chi_1 - \chi_2)(g_i)(\chi_1 - \chi_2)(g_j) \sum_{k\in S} \left(\sum_{h\in S_k} \tau(h) \right) c_{ijk} \right]. \tag{3.1}$$

Proof. As an element of $\mathfrak{C}G$, f is an idempotent affording ψ^G, so using (1.6) we have

$$\begin{aligned}(\chi_1 + \chi_2, \psi^G) &= (\chi_1, \psi^G) + (\chi_2, \psi^G)\\ &= \chi_1(e(\chi_1)f) + \chi_2(e(\chi_2)f)\\ &= (\chi_1 - \chi_2)[(e(\chi_1) - e(\chi_2))f].\end{aligned}$$

Here the last equality follows from the fact that $\chi_i(e(\chi_j)f) = 0$ if $i \neq j$. Now using formula (1.2) for $e(\chi_i)$ and the definition of f we obtain

$$\begin{aligned}(\chi_1 + \chi_2, \psi^G) &= z| G |^{-1} \sum_{g\in G} (\chi_1 - \chi_2)(g) \sum_{h\in H} \tau(h)(\chi_1 - \chi_2)(g^{-1}h)\\ &= z| G |^{-1} \sum_{h\in H} \tau(h) \sum_{g\in G} (\chi_1 - \chi_2)(g)(\chi_1 - \chi_2)(g^{-1}h)\\ &= z| G |^{-1} \sum_{k\in S} \left(\sum_{h\in S_k} \tau(h) \right) \sum_{i,j=0}^{s} (\chi_1 - \chi_2)(g_i)(\chi_1 - \chi_2)(g_j) c_{ijk}.\end{aligned}$$

The last equality above follows from Lemma 1. We note that the terms are zero unless $1 \leqslant i, j \leqslant w$. Now since $(\chi_1 + \chi_2, \psi^G) \neq 0$ we can solve for z, obtaining (3.1).

Formula (3.1) represents an improvement over (2.1) for finding z in the sense that fewer structure constants are required. However, (3.1) requires a knowledge of a primitive idempotent of a subalgebra and the multiplicities of χ_1 and χ_2 in an induced character. In the case of exceptional characters, some such information is often available.

We now proceed to apply Theorem 4 to a family of groups studied extensively by Suzuki. We refer to the following hypothesis as (A).

HYPOTHESIS A. The finite group G has an Abelian subgroup A which is the centralizer of each of its nonidentity elements. Let $H = N_G(A)$, $|A| = n$, $[H : A] = l$, and $w = (n - 1)/l$. Assume $w > 1$.

We summarize some of the major properties of groups satisfying (A).

LEMMA 2. *Let G satisfy* (A). *Then the following statements hold.*

(i) *w is an integer and equals the number of conjugacy classes of G which meet A–$\{1\}$. Let $K_1, \ldots, K_w$ be these "special" classes.*

(ii) *There exist exactly w irreducible characters $\psi_1, \ldots, \psi_w$ of H not having A in their kernels, each of degree l, being induced from nonprincipal (linear) characters $\lambda_1, \ldots, \lambda_w$ of A.*

(iii) *There exist w irreducible characters $\chi_1, \ldots, \chi_w$ of G and a sign $\epsilon = \pm 1$ such that*

$$(\chi_i - \chi_j) \mid H = \epsilon(\psi_i - \psi_j) \qquad \text{and} \qquad (\psi_i - \psi_j)^G = \epsilon(\chi_i - \chi_j). \quad (3.2)$$

These χ_i's, called exceptional characters, all have the same degree z and the differences $\chi_i - \chi_j$ have their support on the classes meeting $A - \{1\}$.

(iv) *There exists a fixed integer m such that*

$$(\chi_i, \psi_j^G) = \begin{cases} m + \epsilon, & i = j \\ m, & i \neq j. \end{cases} \quad (3.3)$$

For a proof of this lemma see [8].

THEOREM 5. *Let G satisfy (A) and let the notation be as in Lemma 2. We may assume $g_i \in K_i \cap A$, $i = 1, \ldots, w$. Assume further that $w \geqslant 3$. Then the degree z of the exceptional characters χ_i, $i = 1, \ldots, w$, is determined by the differences $\psi_i - \psi_j$, the group orders of A and G, and those structure constants having all* *three subscripts associated with special classes. Specifically, let the number u be defined by*

$$u = \sum_{i,j,k=1}^{w} (\psi_1 - \psi_2)(g_i)(\psi_1 - \psi_2)(g_j)(\psi_1 - \psi_3)(g_k^{-1})c_{ijk}. \tag{3.4}$$

Then u is a rational integer divisible by A, the sign ϵ is the sign of u, and

$$z = |G||A|/|u|. \tag{3.5}$$

Proof. Let e_1 be a primitive idempotent in $\mathfrak{C}A$ affording λ_1. Then $\lambda_1{}^H = \psi_1$ implies that e_1 is also a primitive idempotent of $\mathfrak{C}H$ affording ψ_1. Since λ_1 is linear, $e_1 = e(\lambda_1) = |A|^{-1} \sum_{a \in A} \lambda(a)\, a^{-1}$. Letting

$$S_k = K_k \cap A = K_k \cap H$$

we have

$$\sum_{a \in S_k} \lambda_1(a) = \begin{cases} \psi_1(g_k) & k = 1,..., w \\ 1 & k = 0. \end{cases}$$

Applying this information to (3.1), we obtain

$$z = (\chi_1 + \chi_2, \psi_1{}^G)|G| \Big/ \sum_{i,j=1}^{w} (\chi_1 - \chi_2)(g_i)(\chi_1 - \chi_2)(g_j)|A|^{-1} \times \left(1c_{ij0} + \sum_{k=1}^{w} \psi_1(g_k^{-1})c_{ijk}\right).$$

We know by (3.3) that

$$(\chi_1 + \chi_2, \psi_1{}^G) = (m + \epsilon) + m$$

and by Lemma 1,

$$\sum_{i,j=1}^{w} (\chi_1 - \chi_2)(g_i)(\chi_1 - \chi_2)(g_j)c_{ij0} = |G| \|\chi_1 - \chi_2\|^2 = 2|G|,$$

so that

$$z = (2m + \epsilon)|G||A| \Big/ \left[2|G| + \sum_{i,j,k=1}^{w} (\chi_1 - \chi_2)(g_i)(\chi_1 - \chi_2)(g_j)\psi_1(g_k^{-1})c_{ijk}\right]. \tag{3.6}$$

Similarly, starting with an idempotent e_3 affording λ_3, where $\lambda_3{}^H = \psi_3$, and noting that $(\chi_1 + \chi_2, \psi_3{}^G) = 2m$, we obtain

$$z = 2m|G||A| \Big/ \left[2|G| + \sum_{i,j,k=1}^{w} (\chi_1 - \chi_2)(g_i)(\chi_1 - \chi_2)(g_j)\, \psi_3(g_k^{-1})c_{ijk}\right]. \tag{3.7}$$

Since

$$(\chi_1 - \chi_2)(g_i) = \epsilon(\psi_1 - \psi_2)(g_i)$$

and since $\epsilon^2 = 1$, we can replace each $\chi_1 - \chi_2$ by $\psi_1 - \psi_2$ in (3.6) and (3.7). We solve these two simultaneous equations for z, eliminating the unknown m, yielding $z = \epsilon \mid G \mid \mid A \mid / u$. Now since z is a positive integer dividing $\mid G \mid$, all parts of the theorem follow.

Theorem 5 may be obtained by another approach using recent results of Curtis and Fossum [2]. Still assuming Hypothesis A, let $G = \bigcup_{i \in I} Ax_iA$ be an (A, A)-double coset decomposition of G. Let $J \subset I$ be those indices j such that $\lambda_1 = \lambda_1^{x_j}$ on $A \cap A^{x_j}$, where $\lambda^x(x^{-1}ax) = \lambda(a)$. It can be seen that $j \in J$ if and only if $x_j \in A$ or $x_j \notin H$. We may assume that $x_1 = 1 \in A$. Beginning with a result in [2] which gives a formula for $e(\chi_i)\, e_j$, a fairly long calculation of the expression

$$(\chi_1 - \chi_2)[(e(\chi_1) - e(\chi_2))(e_1 - e_3)]$$

yields $z = \epsilon \mid G \mid / u'$ where

$$u' = \sum_{\substack{j \in J \\ j > 1}} \sum_{\substack{a_1 \in A \\ a_2 \in A}} (\chi_1 - \chi_2)(a_1 x_j^{-1})(\chi_1 - \chi_2)(a_2 x_j)(\lambda_1 - \lambda_3)(a_1 a_2).$$

We note that by comparing this with (3.5) we must have $u' = \mid A \mid^{-1} u$, which may be shown by direct computation.

The proof of Theorem 5 was based chiefly on orthogonality properties of idempotents in the group algebra. Since these properties are, for the most part, carried by the orthogonality relations for group characters one should suspect that a purely character theoretic proof exists. This was the case with Theorem 3. The appropriate character theoretic proof of Theorem 5 yields an even stronger theorem, applying to a wider class of groups than those satisfying (A).

THEOREM 6. *Let ψ_1, ψ_2, ψ_3 be three of a family of irreducible characters of the same degree on a subgroup H of G such that the differences $\psi_i - \psi_j$ have support on a* T.I. *set. Let $\{\chi_i\}$ be the associated exceptional characters of G, so that $(\psi_i - \psi_j)^G = \epsilon(\chi_i - \chi_j)$, where $\epsilon = \pm 1$. Then*

$$z = \epsilon \mid G \mid \Big/ \sum_{i,j,k=1}^{w} \mid C_G(g_k) \mid^{-1} (\psi_1 - \psi_2)(g_i)(\psi_1 - \psi_2)(g_j)(\psi_1 - \psi_3)(g_k^{-1}) c_{ijk} \tag{3.8}$$

where z is the common degree of the exceptional characters and $K_1, \ldots, K_w$ are the conjugacy classes of G meeting the T.I. *set.*

Proof. Let D be the denominator in the formula. Then

$$D = \sum_{i,j,k=1}^{w} |C_G(g_k)|^{-1}\epsilon(\chi_1 - \chi_2)(g_i)\,\epsilon(\chi_1 - \chi_2)(g_j)\,\epsilon(\chi_1 - \chi_3)(g_k^{-1})c_{ijk}$$

$$= \epsilon \sum_{k=1}^{w} |C(g_k)|^{-1}(\chi_1 - \chi_3)(g_k^{-1}) \sum_{i,j=1}^{w} (\chi_1 - \chi_2)(g_i)(\chi_1 - \chi_2)(g_j)c_{ijk}$$

$$= \epsilon \sum_{k=1}^{w} |C(g_k)|^{-1}(\chi_1 - \chi_3)(g_k^{-1})(\chi_1 + \chi_2)(g_k)\,|G|z^{-1}$$

$$= \epsilon |G| z^{-1}(\chi_1 - \chi_3, \chi_1 + \chi_2)$$

$$= \epsilon |G| z^{-1}.$$

The third line involves an application of Lemma 1. This proves the theorem.

We note that under the hypotheses of Theorem 5, $|C_G(g_k)| = |A|$ for $k = 1,..., w$. Making this substitution in (3.8) yields (3.5).

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